

hw3

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Problem 1: Simple Mixture EM

Read sections 8.5.1 and 8.5.2 of “The Elements of Statistical Learning”.

Also skim through pages 97-105 of Givens and Hoeting.

Code up Algorithm 8.1 of the Elements.

This is the simple mixture of two univariate normal components.

Get it working on real and simulated data.

Problem 2: Rejection for the $N(0,1)$

Code up the squeezed rejection sampling algorithm for the standard normal.

Check that it works!!

Problem 3: Bayesian Simple Linear Regression

Consider the model:

$$y_i = \beta x_i + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2).$$

Here we assume we know σ .

Let's try assessing prior sensitivity using importance sampling.

Use the normal prior $\beta \sim N(0, 1/A)$ with A small (a “non-informative prior”) as p_1 .

Then let p_2 be a normal prior with a bigger A and compute the expected value and variance of β under the p_2 prior using draws from the posterior from the p_1 posterior and importance sampling.

Check your results against the formulas since we know the answer in this case.

Remember, for

$$Y = X\beta + \epsilon, \quad \epsilon \sim N(0, \sigma^2 I), \quad \beta \sim N(\bar{\beta}, A^{-1}),$$

$$\beta | \sigma, y, X \sim N(\tilde{\beta}, \tilde{A}^{-1}), \quad \tilde{A} = \frac{X'X}{\sigma^2} + A, \quad \tilde{\beta} = \tilde{A}^{-1} \left(\frac{X'y}{\sigma^2} + A\bar{\beta} \right)$$